## Contributions

# Post-Modern Portfolio Theory

**by Pete Swisher, CFP®, QPA, and Gregory W. Kasten, M.D. , CFP®, CPC, AIFA®**

## Executive Summary


- Modern portfolio theory (MPT) and its mean-variance optimization (MVO) model for asset allocation are Nobel Prize-winning theories of global equilibrium, but are unreliable for the primary task to which the financial services industry applies them—building portfolios.
- Post-modern portfolio theory (PMPT) presents a new method of asset allocation that optimizes a portfolio based on returns versus downside risk (downside risk optimization, or DRO) instead of MVO.
- The core innovation of PMPT is its recognition that standard deviation is a poor proxy for how humans experience risk. Risk is an emotional condition—fear of a bad outcome such as fear of loss, fear of underperformance, or fear of failing to achieve a financial goal. Risk is thus more complex than simple variance but can nonetheless be modeled and described mathematically.
- Downside risk (DR) is a definition of risk derived from three sub-measures: downside frequency, mean downside deviation, and downside magnitude. Each of these measures is defined with reference to an investor-specific minimal acceptable return (MAR).
- Portfolios created using MVO and DRO are often similar and the differences in absolute risk and return values small—diversification works regardless of how you measure it. Yet DRO seems to avoid the known errors of MVO and provide a more reliable tool for choosing the "best" portfolio.
- PMPT points the way to an improved science of investing that incorporates not only DRO but also behavioral finance and any other innovation that leads to better outcomes.



*Pete Swisher, CFP®, QPA, serves as vice president of advisory services at Unified Trust Company in Lexington, Kentucky, where he works primarily with institutional clients as a specialist in the fiduciary management of qualified plans and trusts.*

*Gregory W. Kasten, M.D., CFP®, CPC, AIFA®, is a board-certified anesthesiologist who left the practice of medicine to found Unified Trust Company, where he now serves as president and CEO. Dr. Kasten is the author of numerous articles, research papers, and the book Retirement Success.*


"All truth passes through three stages. First it is ridiculed. Second, it is violently opposed. Third, it is accepted as being self-evident."
—*Arthur Schopenhauer*

The perfect investment, as everyone knows, is positively skewed, leptokurtic, and has low semi-variance. But these moments about the mean of an investment's return probability distribution are at least partially incompatible since investment returns are non-Gaussian, and variance/semi-variance obviously loses its utility in non-mesokurtic (that is, non-Gaussian) skewed distributions. Which is exactly the point.

Just in case you're not sure what that first paragraph means, here is a rough translation: people like to make money, not lose it. Making above-average amounts of money frequently is better than a tiny chance at winning the lottery. And when an investment performs poorly, it's best if it doesn't perform *too* poorly or too often. The problem is that while we have an elegant mathematical model for describing the perfect investment—called modern portfolio theory (MPT)—that model is wrong. Not wrong in the sense that the overall theory is no good, just wrong in the specific sense that it produces inefficient (and sometimes silly[1]) portfolios. And we've known it for decades.

The primary reason MPT produces inefficient portfolios (even though the whole point is supposedly the building of *efficient* portfolios) is simple: standard deviation is not risk. Risk is something else, and we need a better mathematical framework to describe it. The primary purpose of this paper is to describe that framework and suggest a use for it—the building of better portfolios through downside risk optimization (DRO). We define DRO as optimization of portfolio risk versus return using downside risk as the definition of risk instead of standard deviation. The secondary purpose of the paper is to give definition to the concept of post-modern portfolio theory (PMPT) and how we as financial planners can apply it on our clients' behalf.

## Background: The Giants of Portfolio Theory

In 1959, Harry Markowitz, the "father of modern portfolio theory," published *Portfolio Selection*,[2] in which he proposed that investors expect to be compensated for taking additional risk, and that an infinite number of "efficient" portfolios exist along a curve defined by three variables: standard deviation, correlation coefficient, and return. The efficient-frontier curve consists of portfolios with the maximum return for a given level of risk or the minimum risk for a given level of return. The algorithm used to generate the curve is known as mean variance optimization (MVO), since what is being optimized is return versus standard deviation, or *variance* from the *mean* return. The work eventually won Markowitz a share of the 1990 Nobel Prize.[3]

One of those he shared the prize with was William Sharpe, who extended Markowitz's work. Sharpe credits Markowitz for taking a personal role in helping shape the doctoral dissertation that led to the capital asset pricing model, or CAPM. In Sharpe's words: "The CAPM is built using an approach familiar to every micro economist. First, one assumes some sort of maximizing behavior on the part of participants in a market; then one investigates the equilibrium conditions under which such markets will clear."[4] Later work from Sharpe gave us the information ratio, a version of which became known as the Sharpe ratio—the first major attempt to create a measure for comparison of portfolios on a risk-adjusted basis.

But MPT and CAPM were not originally created for the purpose to which our industry has adapted them—asset allocation. The theoretical foundation created by Markowitz and Sharpe was an *equilibrium* model that attempts to solve the investment problem for all investors simultaneously.[5] To answer the question of how to build better portfolios—how to optimize—MPT is no longer the best (or only) tool for the job.

## Downside Risk Optimization Defined

The "efficient frontier" software so prevalent today asks a simple question: what is the best return an investor can get for a given level of risk? Conversely, what is the least risk an investor can take for a given level of return? Mean variance optimization attempts to answer these questions using standard deviation as the definition of risk. Downside risk optimization does not change the question—the goal is still the "optimum" mix of risk versus reward—but the definition of risk is different. Instead of standard deviation, DRO uses downside risk.

## Making the Case for DRO

We propose that DRO is superior to MVO as an asset allocation tool. Here are the key arguments in support of that proposition:

1. Standard deviation (SD) can lead to nonsensical results when used as a risk proxy, whereas various downside risk measures (DR) more closely capture the human conception of risk.
2. Even if volatility were a perfect representation of risk, it still would not work perfectly because financial asset returns do not follow a normal distribution.
3. When we put DRO and MVO head to head and compare portfolios, DRO wins. Specifically, DRO outputs make intuitive sense while MVO outputs often do not, and MVO outputs frequently reach risk conclusions opposite those of DRO.

## The Pension Research Institute and Dr. Frank Sortino

Before we move on to the theoretical framework for DRO and present sample data supporting its use, we wish to acknowledge our most important source.

The authors are indebted to the Pension Research Institute in San Francisco, California, and its founder, Dr. Frank Sortino, for much of their understanding of PMPT. *Managing Downside Risk in Financial Markets*, by Frank Sortino and Stephen Satchell, is an anthology of papers on the subject of risk. Study the book, read the papers on the PRI's Web site, surf the Internet for papers going back to the 1970s by Balzer, Bawa, Fishburn, and others, and we challenge you to draw any conclusion other than that DRO is preferable to MVO.

Included with *Managing Downside Risk* is a CD-ROM with source code that permits researchers to build their own downside risk analysis tools. Using this source code we created downside risk optimization software to benefit pension and individual clients of our firm, and that software is the source of the data presented. Since lack of awareness and lack of available software are two of the key reasons why DRO has not yet caught on for mainstream investment advice, we note that several DRO software tools should be commercially available in 2005.

## The Million-Dollar Question: What Is Risk?

"Risk, like beauty, is in the eye of the beholder."
—*Leslie A. Balzer*[6]

We know what risk is not. It is not volatility. "Other things being equal, most investors will prefer less volatile returns to more volatile returns. Other things, however, are not usually equal."[7] It would be nice to avoid volatility, true, but volatility does not describe the complex emotions that equate to our concept of risk.

Investors see risk in three ways:

- The risk of loss (that is, returns below zero)
- The risk of underperformance (returns below a benchmark, such as a neighbor's portfolio or the S&P 500)
- The risk of failing to meet one's goals

Risk is the potential for a bad outcome. Losing money, underperforming, failing to meet financial goals—those are real-life human concerns. Yet this risk definition—potential for an undesirable outcome—is fuzzy. Hard to quantify mathematically. But just because a concept has no clear mathematical equivalent does not mean that we cannot create mathematical models to describe it.

## Isaac Newton and Flying Apples

Imagine that the first draft of Newton's *Principia* treatise on gravity has a flaw—it says that apples fall upward off the tree 50 percent of the time. He gets everything else right, but the formula insists that apples fall upward or downward equally often. The formula therefore fails an important test: it does not describe the reality we observe.

To ensure any theory of risk is supported by observation, we should test any risk definition against how humans actually behave to determine if our definition is accurate. Consider the example in Table 1 of two investments, A and B. An investor has a minimum acceptable return (MAR)—the number his financial planner tells him he needs for long-term success—of 4.50 percent.

TABLE 1

**Does Volatility = Risk?**

| | Investment A | Investment B |
|---|---|---|
| Year 1 Return | 4.50% | 5.00% |
| Year 2 Return | 3.00% | 20.00% |
| Year 3 Return | 3.50% | 33.00% |

Which investment is "riskier"? According to MPT dogma, risk equals standard deviation, making investment

B riskier since it is extremely volatile—a nonsensical result since most investors would clearly identify investment A as the riskier choice. Sortino notes that "M-V is blind" to the fact that all of the variance in investment A occurs at returns below those of investment B.[8] The variance of A is very low, true, but the returns all fall at or below the investor's MAR, while those of B all fall above.

As Brian Rom and Kathleen Ferguson report: "It has long been recognized that investors do not view as risky those returns above the minimum they must earn in order to achieve their investment objectives. They believe that risk has to do with the bad outcomes...and that losses weigh more heavily than gains."[9] Investors are worried about downside deviation, not upside deviation.

Markowitz himself said that "downside semi-variance"[10] would build better portfolios than standard deviation. But as Sharpe notes, "in light of the formidable computational problems...he bases his analysis on the variance and standard deviation."[11] Markowitz did not have a Dell laptop with a 60 Gb hard drive and Microsoft Excel in 1959.

A valid risk definition must not yield nonsensical answers—it must describe actual investor fear of bad outcomes with reasonable accuracy. And just as apples do not fall upward 50 percent of the time, returns above the mean—or MAR—are not what investors fear.

## Mathematical Characteristics of Human Risk Perceptions

Here are the findings of various theorists and experimenters concerning how humans view risk.[12]

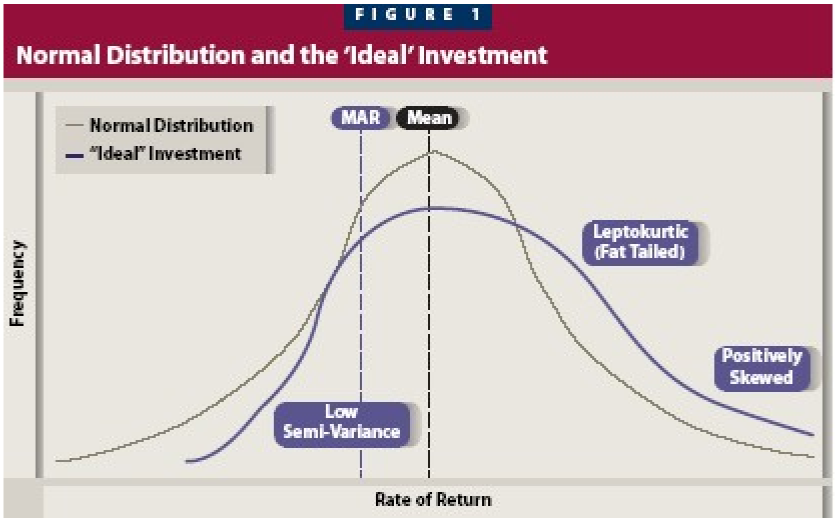
- **Fear of loss is exponential.** Anxiety over a loss increases exponentially as the magnitude of the loss increases.
- **Upside marginal utility leakage.** Happiness over a gain decreases as the magnitude of the gain increases. The investor's utility or "usefulness" for very high returns is not much higher than for merely good returns (there is utility "leakage").
- **Jump discontinuity.** There is a sudden leap in anxiety when returns go below a threshold, such as zero. This is called a "jump discontinuity" in the utility curve because the investor's utility for the returns "jumps" downward when the return is even the smallest fraction below zero (or the investor's minimum acceptable return).
- **Risk is asymmetric.** The way we feel about losses is not the mirror image of how we feel about gains. The shape of the utility curve is different on the left than it is on the right.
- **Risk is situational and investor-specific.** Standard deviation assumes every investor in the world views risk identically. Yet we know that people vary, and that even the same person views risk differently in different scenarios.
- **Risk is relative to a personal benchmark, or MAR.** And that benchmark is not the mean, or average return. Investors have goals they want to achieve and a rate of return that will accomplish those goals. The benchmark, or minimal acceptable return, is therefore an investor-specific "hard target" such as 6 percent or 7 percent. Returns below the MAR are what investors fear.

To build a mathematical definition of risk that is consistent with human behavior, the definition must incorporate the above characteristics to the greatest extent possible.

## DRO Explained: Understanding the Math Without the Math

Mean variance optimization and downside risk optimization both rely on complex math, so an understanding of certain concepts is essential.

**Normal (or mesokurtic, or Gaussian) distribution.** A normal curve is a bell curve (see Figure 1). A distribution is simply the way points on a graph are distributed. A return probability distribution is the curve under which the returns for a given investment fall—the distribution of the probabilities (or frequencies) with which given rates of return occur.




**Standard deviation.** Roughly two-thirds of all data points—all returns—fall within one standard deviation of the mean, or average, return. Over 95 percent fall within two standard deviations, and over 99 percent within three standard deviations.

**Non-normal distribution.** Not a bell curve. The shape of the curve is not symmetrical, which means that upside deviations are not the same as downside deviations. Most financial asset returns follow a non-normal distribution.

**The "perfect investment."** See Figure 1. Here again are the characteristics of the "perfect investment" described in the very first paragraph:

- **Positively skewed.** The "tail" of the distribution points to the right. This means that there are enough very good outcomes to stretch the tail out in a positive direction. By contrast, a negatively skewed distribution might indicate a handful of catastrophic losses. The "coefficient of skewness" is the measure of this characteristic and is also known as the third "moment about the mean" of the return probability distribution.
- **Leptokurtic.** The "fourth moment" is the coefficient of kurtosis, a measure of how "fat" the tail of the distribution is. "Leptokurtic" means that there are more returns spread out along the tail instead of bunched around the mean.
- **Low downside semi-variance.** When returns fall below the mean, they tend not to fall too far below.

## Two Key Reasons

There are two key reasons why standard deviation cannot accurately represent risk.

**Reason #1: non-normal distributions.** Financial asset returns do not follow a normal distribution. As noted above, the distributions are asymmetric, and upside deviation therefore differs from downside deviation, making the use of standard deviation inherently inaccurate.

**Reason #2: mean versus MAR.** Even if financial returns were perfectly symmetrical (and actually they're close—another of the reasons standard deviation has been viewed for so long as a valid risk proxy), standard deviation would still fail to describe human risk. Remember that risk is relative to a personal benchmark, or minimal acceptable return. Since mean and MAR are not the same number, the downside risk (outcomes below the MAR) cannot be symmetrical to the upside (returns above the MAR). Look at Figure 1—even in the normal distribution, the portion of the curve to the left of the MAR line (the downside deviations) is very different from the portion to the right (the upside deviations).

## Putting It All Together: A Formula for Downside Risk

Downside risk equals deviations below the MAR. But how do we measure it? How can we distill it to a single number for use in downside risk optimization software? Sortino offers a formula that combines the following elements:

1. **Downside frequency.** The frequency, expressed as a percentage, of returns below MAR. In a series of 100 months of returns, how many of the returns fell below the monthly MAR? If the MAR is 6 percent a year, then the monthly MAR is .5 percent. If the asset returned less than .5 percent in 35 months out of 100, then the downside frequency is 35 percent.
2. **Average downside deviation.** The average size of the deviation below the MAR.
3. **Downside magnitude.** The worst-case scenario, represented by the return below MAR at the 99th percentile,

We then use a formula to combine these three statistics into a single downside risk measure. The result is expressed as a percentage, much like standard deviation, and the values themselves might even be similar—such as 15 percent for a stock portfolio. But there the resemblance to standard deviation ends, and it is important to note that downside risk cannot simply replace standard deviation in the MVO formulas. An entirely different method of analysis is required.

## The PMPT Toolkit: Replacing Alpha, Beta, and the Sharpe Ratio

The computer age has given virtually every advisor a toolkit that includes data such as $r^2$, beta, alpha, and the Sharpe ratio, and the industry now takes it for granted that these statistics are helpful or even necessary in describing the desirability of an investment. Yet each of these measures relies on standard deviation as its foundation—a foundation we know to be flawed—and we suggest, therefore, that measures such as alpha and beta are for the most part *not useful*. PMPT offers alternatives, including those in Table 2.

TABLE 2

**PMPT Versus MPT Measures**

| Purpose | MPT Version | PMPT Version |
|---|---|---|
| Risk Measure | Standard Deviation | Downside Risk (DR) |
| Outperformance vs. Benchmark | Alpha | Omega Excess;<br>Also Excess Return (Above MAR) |
| Risk Compared to Benchmark's Risk | Beta | DR vs. Benchmark DR (Though Various Betas Could Be Calculated Using DR Components) |
| Excess Return Per Unit of Risk | Sharpe Ratio | Sortino Ratio (Excess Return DR) |

Now that we have established the theoretical framework for downside risk optimization, it is time to present evidence as to why it is preferable to mean variance optimization.

## Building Better Portfolios with DRO

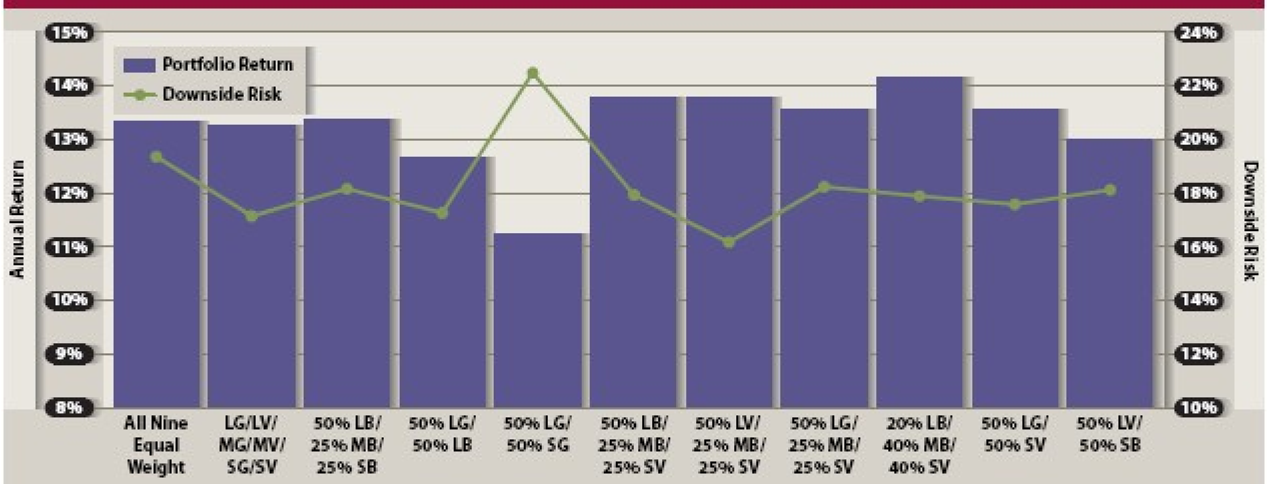
We make this important clarification from the outset: DRO does not make the future any more knowable. The portfolios we build with DRO are not necessarily better performers than portfolios constructed using MVO (though of course we expect them to be, or what would be the point?). The difference is simply that some of the known errors of MVO are not present from the start. As Sortino states in the opening essay of *Managing Downside Risk*, "If a mean-variance optimizer won't give you the right answer when you know what the right answer is...how reliable is it in a complex, realistic situation, where nobody knows what the right answer is?"[13]

With few exceptions, the following data are based on the period from 1984 to 2004. Because the purpose was to compare optimization methods and not to choose the best future portfolio, we deemed it inappropriate to base the analysis on projected future market values as is customary in MVO. We also ran tests over various time periods, including back to 1926, where data are available. We ran rolling five-year periods to help reduce the inherent bias in selecting data from any particular historical period. And, most importantly, we "bootstrapped" the data, a statistical technique that leverages the available data by positing what might have happened—1,000 years of "what ifs"—instead of simply asking what *did* happen.[14] Each of these added steps, though not discussed here, affirms that the generalized results presented for the 1984–2004 period are reasonable.[15]

In Figure 2 on the previous page, we see a comparison of various allocations. Some portfolios did better than others, and some of the better-performing portfolios had lower downside risk as well, which would make them logical choices as "efficient" portfolios.

FIGURE 2

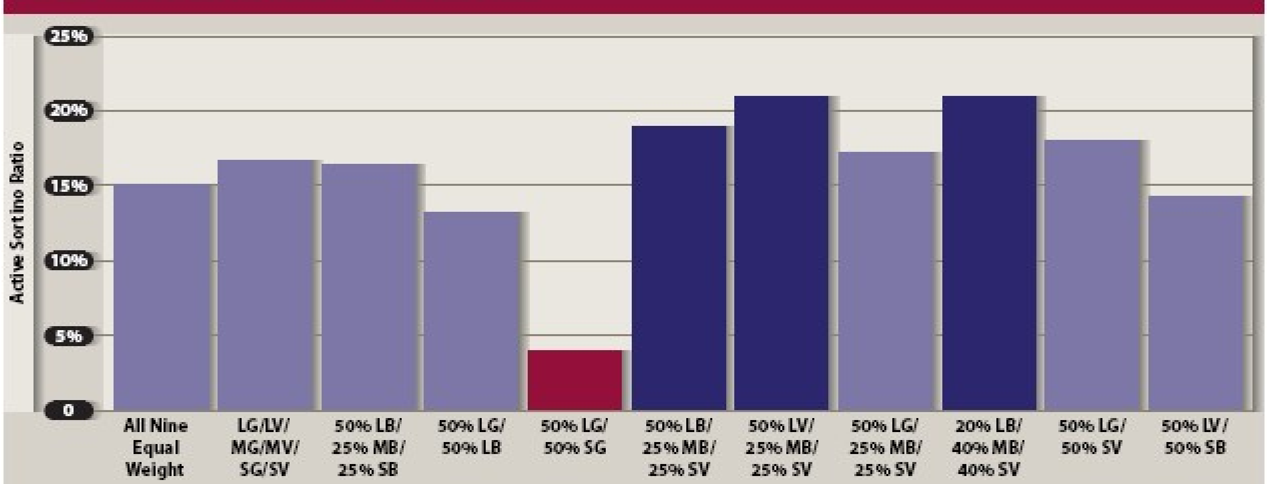
Risk Versus Reward for Various Asset Mixes




In Figure 3 the same portfolios are viewed from the standpoint of excess return per unit of risk—measured by the Sortino Ratio. In this chart we see clearly that an all-growth portfolio was a poor choice, whereas a portfolio heavy on value and on small- and mid-cap stocks offered the best risk-adjusted return.

FIGURE 3

Sortino Ratios: Excess Return ÷ Downside Risk

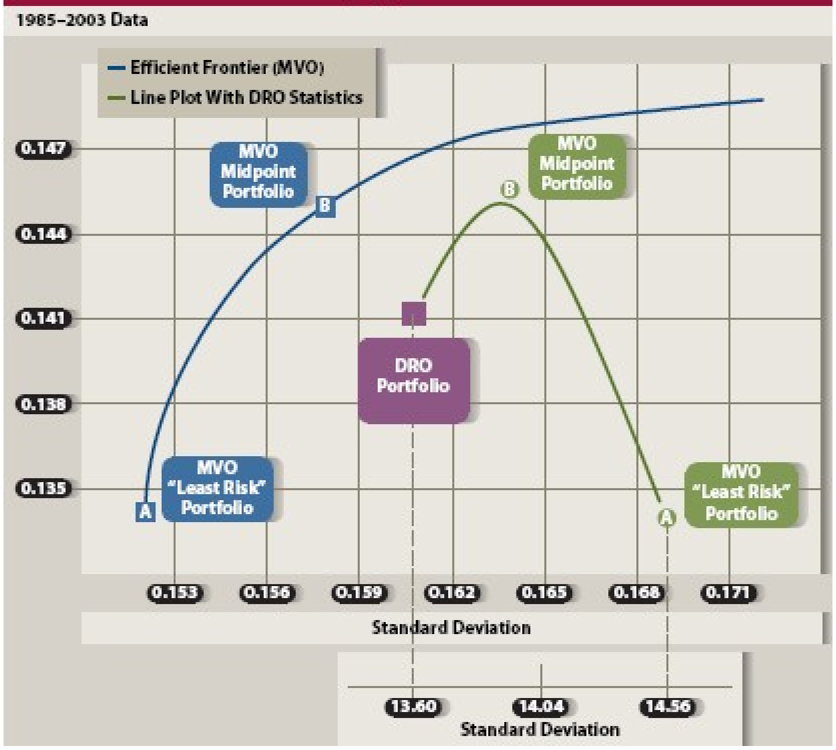



Refining the data obtained from the analysis, we create an all-domestic equity portfolio (DRO portfolio) that delivers the optimum balance of return versus downside risk across various time periods. The critical question, then, is how this same portfolio is judged from the standpoint of MVO. We therefore ran the allocation of the DRO portfolio through a standard mean variance optimizer, and likewise ran several "efficient frontier portfolios" through our downside risk optimizer. The results are summarized in Figure 4 on page 84.

First, a disclaimer: the purpose of Figure 4 is simply to compare the conclusions of the two asset allocation tools. It is actually two charts superimposed, but the charts have different scales and different risk measures. Focus, therefore, on the conclusions as to which portfolios are riskier and not on the obviously different shapes of the curves.

FIGURE 4

**MVO Versus DRO: Same Data, Opposite Results**




As you can see, the mean variance optimizer views the DRO portfolio as sub-optimal—it does not lie on the efficient frontier. We then show the results for a portfolio that lies on the efficient frontier at the midpoint and also the "least risk" equity portfolio according to the efficient frontier. The results are nonsensical: the outputs are clearly incompatible, as seen in Table 3 on page 84.

TABLE 3

**Incompatible Results**

| | Portfolio A: MVO (Least Risk) | Portfolio B: Midpoint of Efficient Frontier | DRO Portfolio |
|---|---|---|---|
| Large Growth | | 10.02% | 8.00% |
| Large Blend | | 17.21% | 10.00% |
| Large Value | 61.52% | | 12.00% |
| Mid Growth | | | 5.00% |
| Mid Blend | | | 25.00% |
| Mid Value | 26.78% | 70.00% | 20.00% |
| Small Growth | | | |
| Small Blend | | | |
| Small Value | | | 20.00% |
| Foreign | 11.70% | 2.77% | |
| Return | 13.33% | 14.44% | 14.11% |
| Standard Deviation | 15.22% | 15.80% | 16.08% |
| Downside Risk | 14.56% | 14.04% | 13.60% |
| Risk Rank With MVO | Lowest | Middle | Highest |
| Risk Rank With DRO | Highest | Middle | Lowest |

The "least risk" portfolio as defined by MVO is actually the most risky of the three portfolios based on DRO. Using MVO, the midpoint portfolio has less risk than the DRO portfolio; using DRO, that outcome is reversed. Additional trials reveal that well-diversified portfolios often yield similar results with both optimizers—note how similar all three portfolios are, based on absolute returns and risk measure—but that the MVO software often yields "extreme" or counterintuitive results. Our conclusion is, therefore, that DRO offers better insight into the optimum portfolio allocation than MVO.

## Why Isn't PMPT Already the Prevailing 'Orthodoxy'?

In retrospect we find it remarkable that DRO is not already the mainstream optimization tool of our industry. As Dr. Sortino says of the efficient frontier software—mean variance optimizers—used by nearly every broker and investment professional in the United States: "They are misleading. They are error maximizers."[16] So why hasn't DRO been mainstream for ten years or more? Dr. Sortino gives the answer by way of the following story:

In 1876, at a celebration of the first centennial of the United States, the American Medical Association sponsored a debate on the merits of sterilization before surgery. A Dr. Joseph Lister (namesake of Listerine® Antiseptic) of Scotland was invited to offer the opposing viewpoint—that sterilization was a good thing. The mainstream "no sterilization" viewpoint, held by the AMA, was argued by then AMA president, Dr. Samuel David Gross.[17] First Lister spoke, presenting statistics and hard evidence on the value of sterilization between patients, such as the reduction in mortality after amputation from 40 percent to 3 percent. Then Dr. Gross spoke. He offered no statistics but instead spoke in reverent terms of his blood-spattered smock and how it reminded him of his sacred duty as a physician. Gross seems to have won, as for years thereafter sterilization and hand washing continued to be openly and "viciously" attacked by the medical establishment in America and Britain, as were Lister and his good friend, Louis Pasteur.[18], [19]

Sortino adds that "the business of providing financial advice is driven by marketing and not technology...the incentive [to change] is not there so long as people are making money." All of the laminated pie charts used as sales pieces work: clients understand them, see plainly that their advisors believe in them, and buy. The tools are well understood, ubiquitous, and inexpensive. This atmosphere creates tremendous momentum for the use of mean variance optimization, which translates to inertia that slows the pace of change.
Beyond DRO: Dynamic Optimization, Behavioral Finance, and a Working Definition of PMPT

Conduct an Internet search under "post-modern portfolio theory" and see what pops up. You will find three veins of thought about the meaning of the term:

1. PMPT describes the formal science of asset allocation using downside risk measures in place of the old MVO methodology. This is the primary use of the term.
2. PMPT emerges from dissatisfaction with the overly clinical application of MPT to the moving target of portfolio allocation—"dynamic asset allocation" being a better approach.
3. PMPT is simply any improvement or enhancement to MPT: in particular, it is whichever improvement or enhancement a particular firm is selling, such as dynamic allocation or portable alpha.

There is a sense that MPT puts us on the wrong track, but with precision, like predicting a 21.4785 percent chance of snow on January 1 of some future year—wrong to four decimal places if it doesn't happen. MPT and MVO give us a theoretical framework that has advanced modern economic thought tremendously, but the investment industry has mass-produced MPT portfolio data to the point of being "wrong to four decimal places." Advisors will sagely note to clients and to one another that a particular mutual fund outperformed its benchmark with less beta and never question whether or not those data mean anything.

The future is uncertain, and so are human beings. Remember the earlier quote from Sharpe: "First, one assumes some sort of maximizing behavior on the part of participants in a market." But behavioral research tells us that market participants do not exhibit maximizing behavior—investors are not rational.[20] Investor irrationality wreaks havoc on attempts to build an equilibrium model, yet it is real—more real and more pertinent to successful investing than the mathematics of global equilibrium. PMPT provides fertile ground for an attempt to integrate behavioral finance into portfolio theory.

Here is a sample of these related veins of thought about PMPT from a presentation by analyst David Nawrocki.[21] "Post-modern portfolio theory...is a closer match to investor behavior..." and "the appropriate response to a nonstationary world is to be adaptive through gradual adjustments to beliefs or expectations...You cannot forecast 20–30 years into the future but you can steer a path through the future with small-to-major adaptations for 20–30 years." By molding the mathematical tools of allocation to fit human behavior and by incorporating the inherently unstable or "nonstationary" nature of reality into portfolio theory, PMPT can provide genuinely useful insights to make us better advisors.

We lay no claim to either the term or the content of post-modern portfolio theory, but we do suggest a loose definition to structure future discussion:

- PMPT is the next step, the successor to MPT
- It involves the use of downside risk optimization in place of mean variance optimization
- It begins the discussion of how to include dynamic allocation and behavioral finance in the optimization problem
- Ultimately, it can and should encompass any idea that advances the science of asset allocation and pricing

If we periodically scrap our entire world view and play the skeptic, challenging even our most basic assumptions, the result will be progress.

## Endnotes


1. For an excellent snapshot of the drawbacks of optimization software, see "Strategic Asset Allocation: Make Love, Not War," by John Rekenthaler, *Journal of Financial Planning*, September 1999. Also see Wealth Management by Harold Evensky (McGraw Hill Companies: New York, 1997), chapter 9.
2. *Portfolio Selection*, Harry M. Markowitz (New Haven, CT: Yale University Press, 1959).
3. For details of the 1990 Nobel Prize in economics and its three winners, go to www.nobelprize.org.
4. From the autobiographical sketch of William F. Sharpe published by the Nobel Prize committee and found on www.nobelprize.org.
5. From a series of conferences in South Africa, Frank Sortino, Pension Research Institute, 1998.
6. "Measuring Investment Risk: a Review," Leslie A. Balzer, *Journal of Investing*, Fall 1994.
7. "Investment Risk: A Unified Approach to Upside and Downside Returns," Leslie A. Balzer, from chapter 8 of *Managing Downside Risk in Financial Markets*.
8. "From Alpha to Omega," Frank Sortino, *Managing Downside Risk in Financial Markets*, p. 5.
9. "A Software Developer's View: Using Post-Modern Portfolio Theory to Improve Investment Performance Measurement," chapter 5 of *Managing Downside Risk in Financial Markets*.
10. *Portfolio Selection*, Harry M. Markowitz (New Haven, CT: Yale University Press, 1959).
11. "A Software Developer's View: Using Post-Modern Portfolio Theory to Improve Investment Performance Measurement," chapter 5 of *Managing Downside Risk in Financial Markets*.
12. Condensed from multiple essays in *Managing Downside Risk in Financial Markets*.
13. "From Alpha to Omega," Frank Sortino, *Managing Downside Risk in Financial Markets*, p. 8.
14. For more information on bootstrapping, see "The Mathematician's View: Modelling Uncertainty with the Three Parameter Lognormal," Hal Forsey, *Managing Downside Risk in Financial Markets*, chapter 4.
15. For additional information on the full study, see "Asset Allocation Optimization Using Downside Risk Analysis," Gregory W. Kasten, 2004, at www.UnifiedTrust.com.
16. Dr. Frank Sortino in an interview with Pete Swisher, November 9, 2004.
17. From Virtual American Biographies at www.famousamericans.net.
18. "Dr. Joseph Lister: Medical Revolutionary," *The Deep Cove Crier*, January 1998.
19. Story related by Dr. Frank Sortino in an interview with Pete Swisher, November 9, 2004.
20. *Retirement Success*, Gregory W. Kasten, chapter 4. See also "The Investor Questionnaire: A Political Necessity, a Practical Failure," Frank A. Sortino, Pension Research Institute, www.sortino.com.
21. "In Search of the Philosopher's Stone," David Nawrocki, SmithBarney Consulting Group, delivered

at Villanova University, 2003.

## Recommended Web Sites

- www.sortino.com: The Pension Research Institute and Dr. Frank Sortino
- www.InvestmentTechnologies.com